Bull and Bear Markets During the COVID-19 Pandemic

John M. Maheu[1] Thomas H. McCurdy[2] Yong Song[3]

[1]DeGroote School of Business, McMaster

[2]Rotman School of Management, University of Toronto

[3]University of Melbourne

CIR 2022 Risk Management Conference, August 10-12, 2022

Discussions about Trends and Cycles in Equity Returns

Positive and negative trends are typically labelled as 'bull' and 'bear' market regimes

Market trends and B&B regime changes are popular topics of discussion

Sample of discussions in the financial press in 2020:

- The Stock Market Is Back in a Bull Market. That Doesn't Mean the Bear Is Over. Barron's, April 7
- Australian Stocks Enter Bull Market After Surging 21% from Low. Bloomberg, April 14
- Bear market rally or new bull? CNBC, May 23
- Is this A Bear Market Rally? Forbes, June 11
- Is this REALLY a Bull Market? stocknews.com, July 4

Can typical models of market phases answer these questions?

Traditional Bull and Bear Dating Algorithms

Identified based on an *ex post* assessment of peaks and troughs in the market index

- Pagan-Sossounov adaptation of Bry-Boschan business cycle algorithm
  - Identify the peaks and troughs by using a window of 8 months.
  - Enforce alternation of phases
  - Eliminate phases less than 4 months unless changes exceed 20
  - Eliminate cycles less than 16 months
- Lunde and Timmermann approach
  - Use a 6-month window to locate the initial local maximum or minimum.
  - Bear market begins with a 20% or more decline
  - Bull market begins with a 20% or more increase

What is Missing?

- *Ex-post* dating schemes fall short in being able guide decisions about regime changes
  - Based on deterministic rules
  - Assume cycles are observable
  - Do not provide probability estimates to guide decisions
  - Cannot make forecasts
- Existing methods of partitioning regimes did not identify intra-regime dynamics that could be useful information for forecasts and decisions
  - Negative sub-trends in a bull market (bull corrections) or
  - Positive sub-trends in bear markets (bear rallies)

Can we identify and forecast stochastic trends and cycles in aggregate equity returns in ways that are useful for investment & risk managment decisions?

TODAY: I will motivate our probabilistic market phase model as a mixture distribution

- discuss the model's performance in capturing periods of significant regime change
- to improve risk management and investment decisions

Motivation for Combining Models

A finite mixture distribution is a mixture of two or more probability distributions

- Variables are drawn from more than one distribution to create a new distribution
- For example: Markov-Switching (MS) Models are mixtures of discrete states
  - Estimated transition probability matrix provides the mixing parameters
  - Early example of regime-change models for interest rates estimated as MS processes

Some benefits of mixing multiple distributions and estimating transition parameters:

- Recognition of the limited domain of particular models, e.g. good versus bad times may involve very different decisions and outcomes
- Identifying the most appropriate model for the various ranges of potential outcomes is a way of managing model risk associated with forecasts
- Features of the component distributions improve estimates of the aggregate mixture

Example applications of this mixture distribution approach from some other research projects:

- Duration-Dependent MS applied to dynamics of GDP, RV, etc.
- MS-ARCH mixtures applied to high versus low volatility regimes
- CPN mixture of jumps and diffusive vol to capture dynamics in the tail of the distribution
- Mixture of sub-models for different time periods: probabilistic model of structural breaks

This Project: A 4-State Model for Market Return Dynamics

We propose a nonlinear model to generate probability estimates for market states and distribution forecasts for aggregate stock returns

- We use a mixture distribution: 4 latent states & a sparse transition probability structure
- Include bear rallies and bull corrections to capture heterogeneous intra-regime dynamics
  - Allows transitions from a bear rally either to a bull or back to the bear state
  - Allows transitions from a bull correction to a bear or back to the bull state
- Provides a full probability model of stock market phases
  - The state-specific distributions provide useful information
  - The mixture governs the market dynamics
- Can forecast (market states, risks, and returns) out-of-sample
  - Computation of the expected return takes all 4 possible future states into account
  - State dependent means and other parameters are components of those calculations
  - Conditional VaR predictions are sensitive to market regimes
- Bayesian estimation accounts for parameter and regime uncertainty

Parameterization of the Restricted MS-4 Model

MS-4

$$
\begin{aligned}
r_t | s_t &\sim N(\mu_{s_t}, \sigma^2_{s_t}) \\
p_{ij} &= p(s_t = j | s_{t-1} = i), \quad i = 1, ..., 4, \quad j = 1, ..., 4. \\
\text{Transition matrix } P &= \begin{pmatrix} p_{11} & p_{12} & 0 & p_{14} \\ p_{21} & p_{22} & 0 & p_{24} \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & p_{43} & p_{44} \end{pmatrix}
\end{aligned}
$$

- States refer to s_t and are identified by:

$$
\begin{aligned}
\mu_1 &< 0 \text{ (bear state)}, \\
\mu_2 &> 0 \text{ (bear rally state)}, \\
\mu_3 &< 0 \text{ (bull correction state)}, \\
\mu_4 &> 0 \text{ (bull state)}; \\
\sigma^2_{s_t} & \quad \text{No restriction}
\end{aligned}
$$

- Regimes combine states: $s_t = 1, 2$ bear regime and $s_t = 3, 4$ bull regime
- State transition restrictions
  - Cannot transition from the bear states to a bull correction
  - Cannot transition from the bull states to a bear rally

Data

- Daily rate of change in the S&P index: February 1885 - November 27, 2020
- Convert to continuously compounded returns (expressed as %)
- Compute weekly continuously compounded return (Wed to Wed)
- Compute weekly RV_t as sum of intra-week daily squared returns

Table: Weekly Return Statistics

N	Mean	$RV^{.5}$	Skewness	Kurtosis
7064	0.125	1.938	-0.568	8.007

Bayesian Estimation of a K-State MS Model

MS-K

$$
\begin{aligned}
r_t|s_t &\sim N(\mu_{s_t}, \sigma^2_{s_t}) \\
p_{ij} &= p(s_t = j|s_{t-1} = i), \quad i = 1, ..., K, \;\; j = 1, ..., K.
\end{aligned}
$$

- 3 groups of parameters:
  - $M = \{\mu_1, ..., \mu_K\}$, $\Sigma = \{\sigma^2_1, ..., \sigma^2_K\}$ and the elements of the transition matrix P
- Given the the parameters $\theta = \{M, \Sigma, P\}$ and the data $I_T = \{r_1, ..., r_T\}$
- Augment the parameter space to include the states $S = \{s_1, ..., s_T\}$

Note that:

- The predictive density and predictive mean are key for forecasting
- Computation of the expected return takes all 4 possible future states into account
- State dependent means and other parameters are components of those calculations

Estimation Details: MMS-JBES-2012

MS-4-State Model Posterior Estimates

Table: Posterior Estimates

	mean	95% DI
bear μ_1	-0.94	(-1.09, -0.79)
bear rally μ_2	0.23	(0.14, 0.32)
bull correction μ_3	-0.11	(-0.21, -0.02)
bull μ_4	0.52	(0.42, 0.64)
σ_1	5.60	(5.21, 6.03)
σ_2	2.44	(2.27, 2.61)
σ_3	1.85	(1.69, 2.04)
σ_4	1.09	(0.97, 1.21)
μ_1/σ_1	-0.17	(-0.20, -0.14)
μ_2/σ_2	0.10	(0.06, 0.13)
μ_3/σ_3	-0.06	(-0.12, -0.01)
μ_4/σ_4	0.49	(0.35, 0.65)

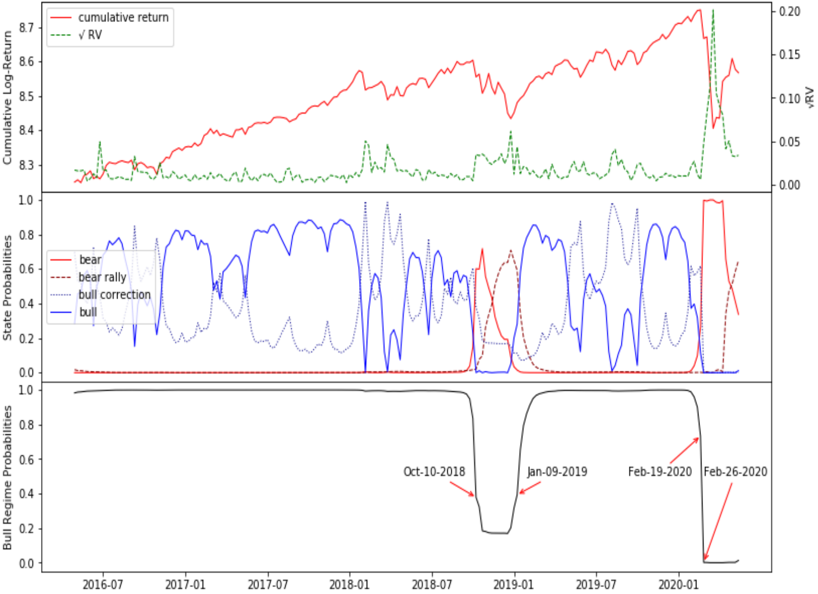
Transition matrix $$\boldsymbol{P} = \begin{pmatrix} 0.906 & 0.092 & 0 & 0.002 \\ 0.013 & 0.968 & 0 & 0.019 \\ 0.013 & 0 & 0.891 & 0.097 \\ 0.001 & 0 & 0.122 & 0.876 \end{pmatrix}$$

Unconditional State Probabilities

	mean
bear π_1	0.084
bear rally π_2	0.245
bull correction π_3	0.356
bull π_4	0.316

- Unconditional prob of bear $\pi_1 + \pi_2 = 0.329$
- Unconditional prob of bull $\pi_3 + \pi_4 = 0.671$

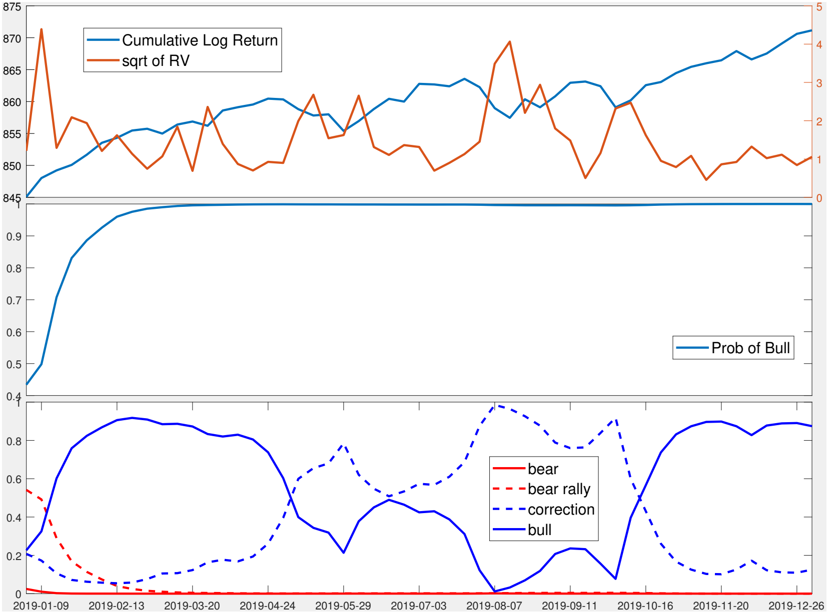
Smoothed Probability Estimates, May 2016-May 2020



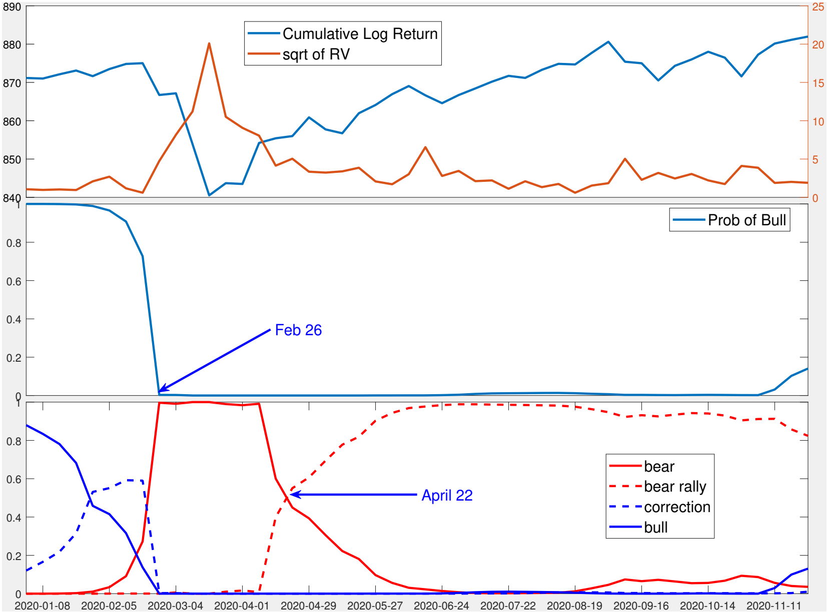
Weekly Stats, Feb-May 2020

				latent state probabilities			
week end	wk ret	wk vol	YTD ret	bear	bear rally	bull correction	bull
1/2/2020	0.55%	1.06%	0.55%	0.000	0.000	0.171	0.829
1/8/2020	-0.15%	0.97%	0.40%	0.000	0.000	0.218	0.781
1/15/2020	1.11%	1.03%	1.51%	0.001	0.000	0.244	0.755
1/22/2020	0.98%	0.96%	2.49%	0.003	0.001	0.352	0.645
1/29/2020	-1.47%	2.09%	1.03%	0.011	0.001	0.616	0.373
2/5/2020	1.86%	2.70%	2.88%	0.040	0.001	0.562	0.398
2/12/2020	1.33%	1.17%	4.22%	0.097	0.002	0.581	0.320
2/19/2020	0.20%	0.61%	4.41%	0.265	0.002	0.618	0.115
2/26/2020	-8.30%	4.74%	-3.89%	0.998	0.000	0.002	0.000
3/4/2020	0.44%	8.16%	-3.45%	0.994	0.005	0.001	0.000
3/11/2020	-13.26%	11.21%	-16.71%	1.000	0.000	0.000	0.000
3/18/2020	-13.38%	20.10%	-30.09%	1.000	0.000	0.000	0.000
3/25/2020	3.18%	10.51%	-26.91%	0.987	0.013	0.000	0.000
4/1/2020	-0.20%	9.07%	-27.11%	0.982	0.018	0.000	0.000
4/8/2020	10.72%	8.05%	-16.40%	0.996	0.005	0.000	0.000
4/15/2020	1.21%	4.14%	-15.19%	0.657	0.342	0.000	0.001
4/22/2020	0.57%	5.04%	-14.62%	0.528	0.470	0.000	0.001
4/29/2020	4.89%	3.35%	-9.73%	0.487	0.512	0.002	0.000
5/6/2020	-3.15%	3.23%	-12.88%	0.413	0.586	0.002	0.000
5/13/2020	-1.00%	3.39%	-13.88%	0.338	0.649	0.002	0.011

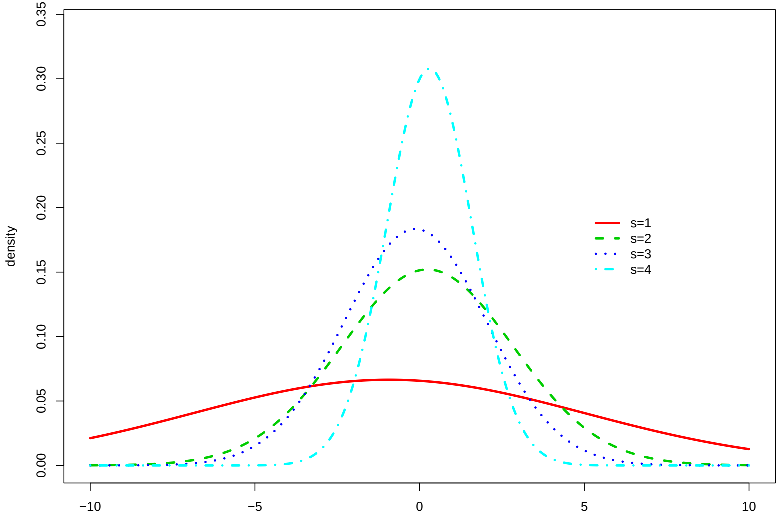
Smoothed Estimates 2019



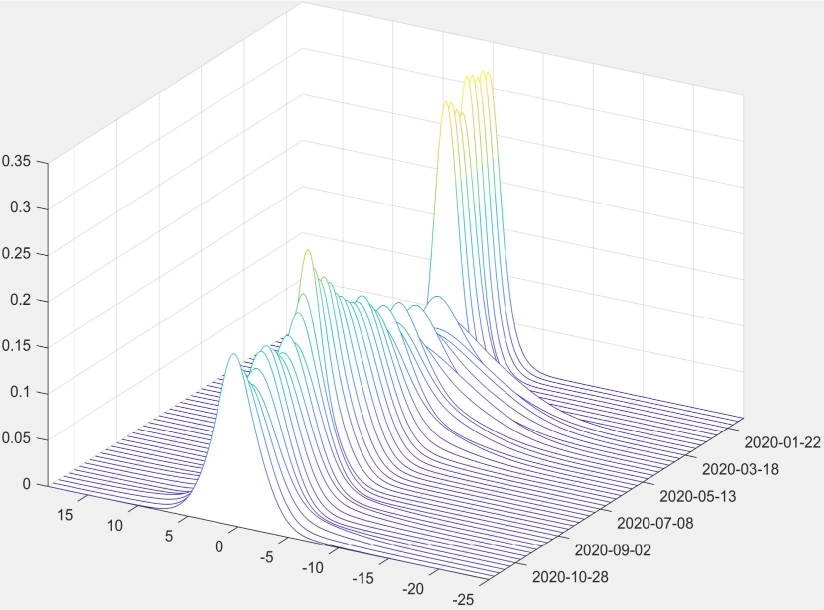
Smoothed Estimates 2020



Distributions for Alternative Market States



Out-of-sample: Predictive Density 1 week ahead



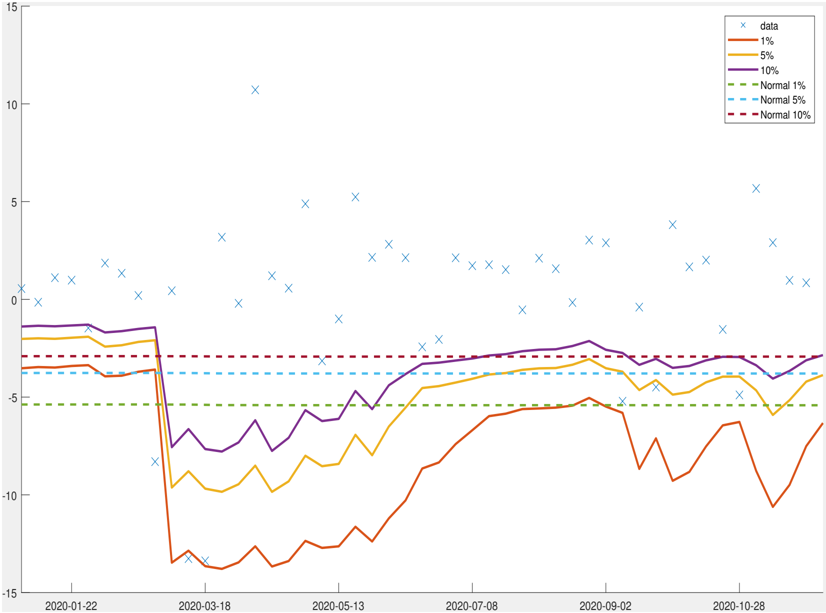
Value-at-Risk (VaR)

- $\text{VaR}_{(\alpha),t}$ is 100α percent quantile for the distribution of r_t given I_{t-1}.
- Compute $\text{VaR}_{(\alpha),t}$ from the predictive density MS-4 model as

$$p(r_t < \text{VaR}_{(\alpha),t} | I_{t-1}) = \alpha.$$

- Given a correctly specified model, the prob of a return of $\text{VaR}_{(\alpha),t}$ or less is α.
- Comparison with $N(0, s^2)$ where s^2 is the sample variance using I_{t-1}.

Out-of-sample: Value-at-risk 1 week ahead



Investment Returns in 2020

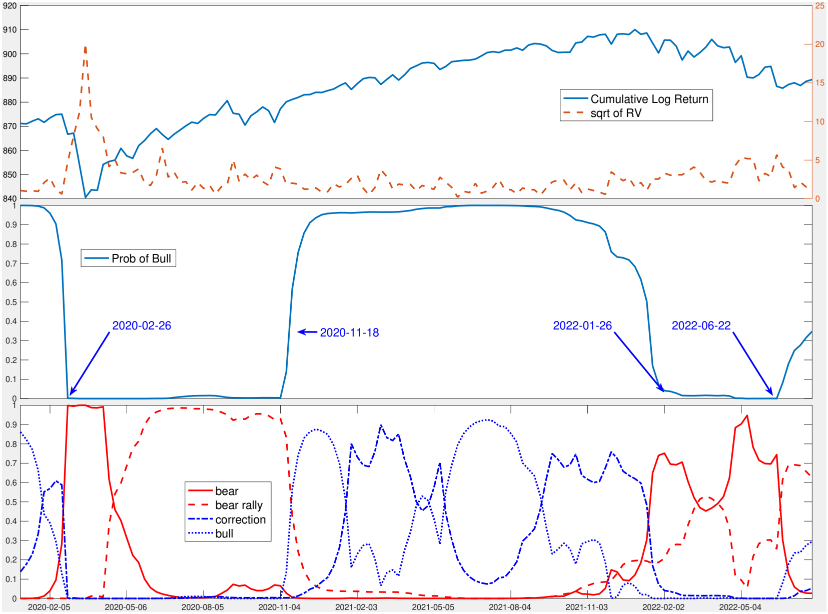
	Return	Sharpe Ratio	number of transactions
Strategy B[a]: $\tau_B = 0.5$	-0.009	-0.048	3
Strategy S[b]: $\tau_S = 0.5$	0.220	1.203	4
Buy-and-hold	0.131	0.566	0

The returns are annualized. Transaction costs are 0.3 cents per share.

[a] Buy if $P(B_t = 2 \mid r_{1:t-1}) > \tau_B$ and sell otherwise.

[b] Buy if $P(s_t = 2 \mid r_{1:t-1}) > \tau_S$ or $P(s_t = 4 \mid r_{1:t-1}) > \tau_S$. Sell otherwise.

Probabilities Updated to July 27, 2022



Summary

- Parameterized a 4-state Markov-switching (MS) model for stock returns
  - Offers richer characterizations of market dynamics
  - Two states govern the bear regime
  - Two states govern the bull regime
  - Intuitive restrictions on the state transition probabilities improves forecasts
- Heterogeneous regimes and intra-regime dynamics
  - Allow for bear rallies and bull corrections without a regime change
  - Most regime turning points occur through bear rally or bull correction
  - Volatility dynamics within regimes is important for transitions

    investors care about both risk and return
- Asymmetric transitions both within regimes and between regimes
  - Bull corrections revert to bull more often than bear rallies bounce back to bear
  - Realized bull and bear regimes can be very different over time
- Probability statements on regimes and future returns available
- Bull corrections and bear rallies empirically important
- Realized bull and bear regimes can be very different over time
- VaR predictions sensitive to market regimes
- July 27, 2022 1-week-ahead forecast for state probabilities
  - bear .032, bear rally .615, bull correction .084, bull .269

Published Paper for additional details: MMS-FRL-2021